Perfect Linear Frequency Modulation of Voltage Controlled Oscillator

Seong-Heon Seo[1, a)]

National Fusion Research Institute, Daejeon 34133, Korea

(Dated: 6 February 2017)

Reflectometer is a frequency modulated continuous wave (FMCW) radar specialized for the fast and broad frequency modulation (FM). A solid state voltage controlled oscillator (VCO) is widely used to linearly modulate the frequency. As the frequency sweep range and rate are increased, the perfect linearization of FM becomes difficult due to the finite response time of electronic components. The first step for linearization is to measure the instantaneous frequency as the frequency is fast swept. The second step is to remove any nonlinearity from the measured frequency trace. The perfect linear FM is achieved by modeling the VCO input circuit as a low pass filter and numerically simulating the circuit behaviour. The simulated VCO input voltage waveform is drived by using an arbitrary waveform generator. The techniques for frequency measurement and frequency linearization are described in detail and the results are presented.

[a)]Electronic mail: shseo@nfri.re.kr

I. INTRODUCTION

Reflectometers[1–3] have been routinely used to measure the plasma density profile in many magnetic fusion devices. A reflectometer can be considered as a frequency modulated continuous wave (FMCW) radar[4] specialized for the fast and broad frequency modulation (FM). The plasma density profile can be reconstructed once the group delay, which is experienced as a microwave propagates through the plasma and then reflects back from the plasma cutoff density[5], is measured as a function of microwave frequency. Since a linear FM gives a simple relation between the group delay and the IF frequency of mixer, a FMCW reflectometer measures the IF frequency of mixer as transimitting and receiving the microwave of which frequency is linearly modulated. The frequency should be swept fast so that the plasma profile remains fixed during the frequency sweep time because various kinds of fluctuation exist in the plasma. The plasma cutoff frequency is in the range of 0-170 GHz in the most fusion plasma experiments depending on the magnetic field strength and the plasma density. In KSTAR reflectometer[6], the microwaves in the frequency range of Q band (33-50 GHz), V band (50-75 GHz), and W band (75-110 GHz) are transmitted through horn antennas installed inside of the vacuum vessel. The whole frequency band is swept in 20 μs. Recently, a frequency sweep time of 2 μs is reported in Tore-supra reflectometer[7].

Voltage controlled oscillator (VCO) is a key component in FMCW radar. A commercially available solid state VCO has a capability to sweep the frequency range even in K band (18-26.5 GHz) within 1 μs. Frequency sweep of much higher band can be implemented by cascading frequency multipliers and amplifiers to the VCO output. A VCO is normally designed to have a linear relationship between the output frequency and the tuning voltage. However the linearity is not perfect. An arbitrary waveform generator (AWG) is used to finely adjust the tuning voltage waveform to get a more linear FM in reflectometers. The voltage waveform of AWG is determined by mathematically inverting the output frequency graph measured versus the tuning voltage. This frequency characteristics of VCO can be statically measured by using a spectrum analyzer. However the VCO's behaviour starts to be deviated from the static characteristics due to the finite response time of electronic components as the frequency sweep rate is increased. For example, the operational amplifier in VCO driver has a finite skew rate and the equivalent circuit of VCO input interface comprises a low pass filter. Therefore there is a time delay before the expected frequency is generated when the tuning voltage is fast swept. This delay should be carefully considered to realize a perfect linear FM.

A perfect linear FM implies a constant slope in the time trace of frequency. Any wiggles deviated from a constant slope indicate that the frequency is nonlinearly modulated. The perfect linearization is first tried by assuming a constant time delay. By optimizing the delay time, the wiggles can be minimized but not completely removed. A more realistic model is tried by simulating the behaviour of the low pass filter in the VCO input interface. This makes the wiggles much more reduced. The wiggles might be further reduced by sophisticating the circuit model. However there is a trick to achieve a perfect linearization without improving the circuit model. Since the above simulation method gives a new AWG voltage waveform resulting in a more linear FM, the method can be repeated with the new voltage waveform and the corresponding frequency measurement. A perfect linear FM is obtained by repeating the above procedure 3-4 times. In this paper, the procedure for the perfect linear FM of VCO is explained in detail. The instantaneous frequency measurement technique is explained in Sec. 2. The frequency linearization method and the results are presented in Sec. 3.

II. INSTANTANEOUS FREQUENCY MEASUREMENT

The frequency measurement is the first step to control the VCO's output frequency. It is not straightforward to measure the instantaneous frequency while the frequency is fast swept. However the time derivative of frequency can be measured by using interferometry[8]. It utilizes the fact that the mixer IF frequency is the frequency difference between two signals fed to the mixer LO and RF inputs. Finally the derivative is integrated to obtain the frequency. An interferometer is constructed as shown in Fig. 1. The IF output is recorded by a 100 MSamples/s digitizer. The IF frequency is digital signal processed through a sophisticated phase detection algorithm based on wavelet transform[9–11]. A frequency modulated signal is divided through a power divider and these two signals are fed to the mixer through coaxial cables of different length. Since there is no frequency dispersion in a coaxial cable, the transit time is only proportional to the cable length regardless of the frequency and the time derivative of frequency. Therefore the mixer IF frequency is proportional to the time derivative of the VCO frequency if the transit time is sufficiently

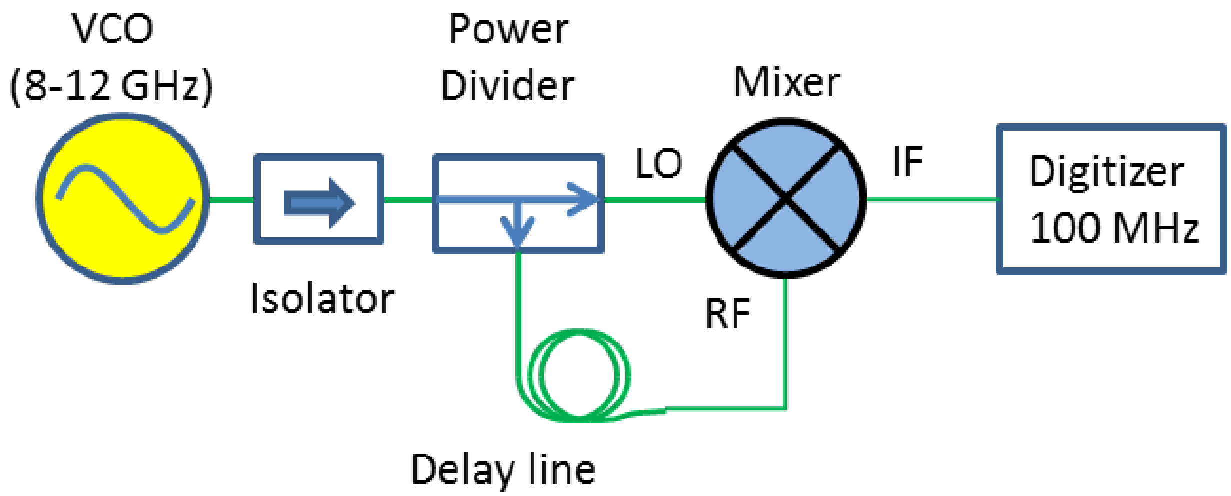

FIG. 1. An interferometer is assembled to measure the instantaneous frequency of VCO

short so that the following approximation is valid.

$$f_{IF} = \int_0^{\tau} \frac{df}{dt} \approx \tau \frac{df}{dt}, \tag{1}$$

where τ is the transit time difference of two cables given by Eq. (2).

$$\tau = \frac{\Delta L}{v_{cable}}, \tag{2}$$

where ΔL is the cable length difference and v_{cable} is the wave propagation velocity inside of coaxial cable which is determined by the dielectric constant. Therefore the mixer IF frequency can be integrated to get the time trace of VCO instantaneous frequency.

$$f(t) = f_0 + \int_0^{t} \frac{f_{IF}}{\tau} dt, \tag{3}$$

where f_0 is the initial frequency. These two parameters τ and f_0 can be experimently determined by noticing the fact that the initial and final frequencies approach the static values measured by spectrum analyzer if a constant voltage is kept long enough before and after frequency sweep.

III. LINEAR FREQUENCY MODULATION

A SMA connectorized version of VCO (VO3262P/00, Sivers IMA) in the frequency range of 13.5-20 GHz is used in the V band KSTAR reflectometer. The output voltage range of

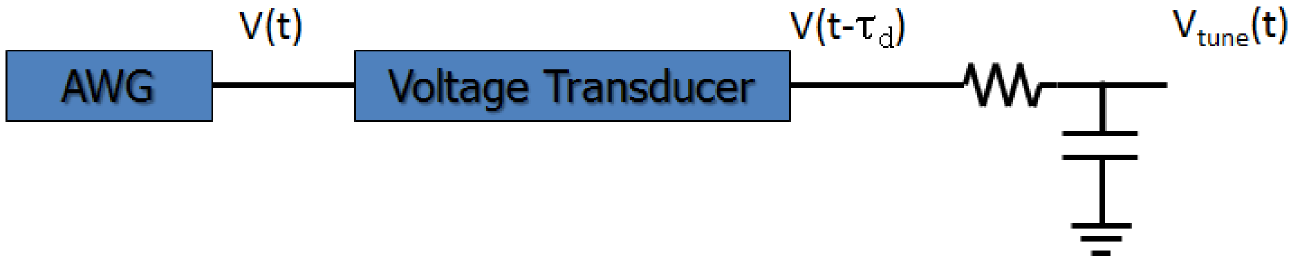

FIG. 2. Block diagram of VCO driver and VCO input interface.

a commercial AWG (ZT530, ZTEC Instruments) is ±10 V but the tuning voltage range of the VCO is 0-20 V. Hence a voltage transducer is needed to drive the VCO by the AWG. The voltage transducer is homemade based on LF356N operational amplifier. The block diagram of AWG, voltage transducer, and the VCO input interface built inside of VCO is drawn in Fig. 2. The circuit model of VCO input interface is taken from the VCO's data sheet[12]. Due to the finite response time of the voltage transducer and the RC low pass filter inside of the VCO, the static characteristics of VCO gets deviated as the frequency sweep rate is increased.

The simplest model of Fig. 2 is to assume that there is a constant time delay of τ_d between the application of a tuning voltage and the appearance of the frequency corresponding to the statically measured VCO characteristics. By inverting the frequency graph with the delay time of τ_d, the tuning voltage waveform is obtained which would linearize FM. The resulting frequency trace is shown in Fig. 3 (a). The frequency linearization is tried by varying the delay time in the time step of 0.02 μs. The time derivative of frequency is a flat line in average but shows many wiggles. Depending on the delay time, wiggles are reduced but not completely removed. The wiggles are minimized for $\tau_d = 0.22$ μs (green line).

A more realistic model is tried. The RC low filter in the VCO input is modeled as in Eq. (4). The delay time of voltage transducer is assumed as a constant as before.

$$V(t-\tau_d) = V_{tune}(t) + RC\frac{dV_{tune}(t)}{dt} \tag{4}$$

For a given AWG voltage waveform, the tuning voltage waveform V_{tune}(t) behind the filter is simulated by solving Eq. (4) numerically. The VCO characteristics is obtained by measuring the output frequency versus the simulated waveform V_{tune}(t). A new linearizing waveform of V_{tune}(t) is obtained by inverting this frequency graph. The actual AWG voltage waveform is calculated by substituting the obtained V_{tune}(t) to the right hand side of Eq. (4). The

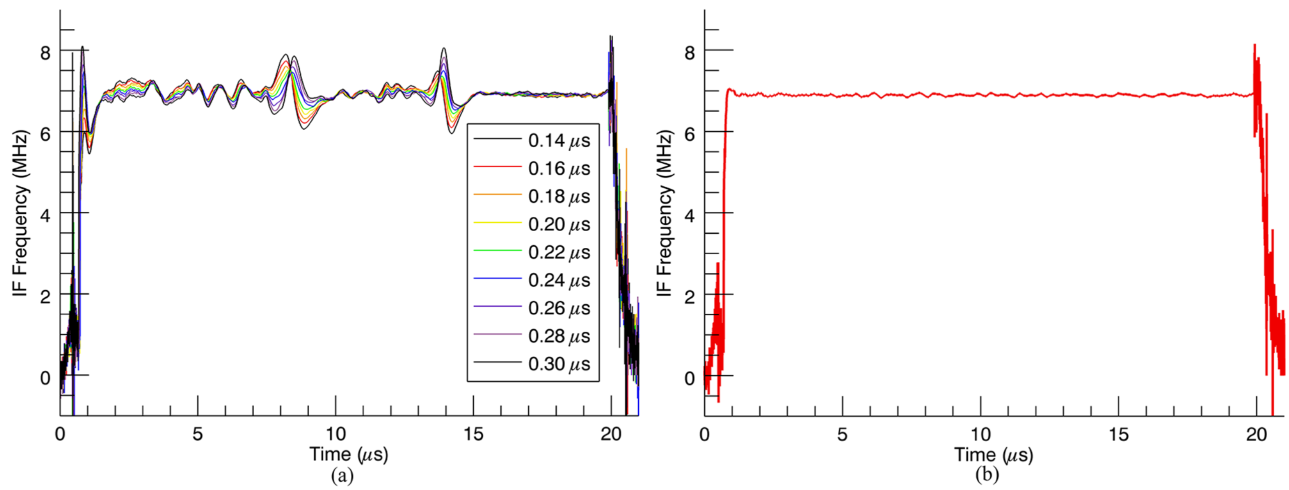

FIG. 3. (a) linearization by a constant delay time model. (b) linearization by a low pass filter model

frequency trace is measured with this new AWG voltage waveform. The role of the above procedure is to transform a set of AWG voltage waveform and the corresponding frequency measurement to a more improved set. By first applying this procedure, the wiggles are much more reduced but still not completely. The best result is obtained with the delay time of 0.19 μs and the RC time of 0.32 μs.

To reduce the wiggles further, a more sophisticated model such as a second order differential equation may be needed. However, there is a trick to reduce the wiggles with the first order differential equation of Eq. (4). In general, a higher order term becomes significant when the variation is large. Therefore, although Eq. (4) is not an exact circuit model, it could give the correct AWG voltage waveform if the nonlinear distortion is sufficiently small. Fortunately the new set obtained by applying Eq. (4) gives much less wiggles. Therefore, it is expected that the wiggles might be further reduced if this procedure is repeated with the improved AWG waveform and the corresponding frequency measurement.

When the second term of the right hand side of Eq. (4) is omitted, that is with the simplest model of constant delay time, the repetition does not eliminate the wiggles. But a few repetitions with Eq. (4) result in almost perfect linear FM as shown in Fig. 3 (b). Therefore Eq. (4) is the lowest order equation which converges to a true linear FM by iteration.

IV. SUMMARY AND DISCUSSION

The FM of VCO is perfectly linearized by applying Eq. (4) interatively. However the frequency of reflectometer is not linearly modulated by linearizing the frequency of VCO because a frequency multiplier is cascaded to the VCO to generate higher frequency range of Q band (30-50 GHz), V band (50-75 GHz), and W band (75-110 GHz). The response time of frequency multiplier can be simply included by changing the time constants of Eq. (4) and then the same procedure can be repeated to accomplish the linearization. In this case, the reflectometer itself is used as an interferometer for frequency measurement by reflecting the microwave from the inner wall of the vacuum vessel. An issue remains that higher frequency components are connected through waveguide and a waveguide has frequency dispersion. This is a problem especially in KSTAR reflectometer. It uses WR-28 waveguide to transfer the microwave to the remotely located antenna. The waveguide is just slightly over sized compared to the fundamental waveguides of Q band and V band, that is WR-22 and WR-15, respectively. However this issue is out of the scope of this paper. It will be the future subject of KSTAR reflectometer.

ACKNOWLEDGMENTS

This work has been supported by the MSIP of Korea under the KSTAR project, and supported by NRF of Korea under basic fusion R&D programs.

REFERENCES

[1] J. L. Doane, E. Mazzucato and G. L. Schmidt, Rev. Sci. Instrum. **52**, 12 (1981).

[2] A. Silva, M. E. Manso, L. Cupido, M. Albrecht, F. Serra, P. Varela, J. Santos, S. Vergamota, F. Eusebio, J. Fernandes, T. Grossmann, A. Kallenbach, B. Kurzan, C. Loureiro, L. Meneses, I. Nunes, F. Silva and, W. Suttrop, Rev. Sci. Instrum. **67**, 4138 (1996).

[3] K. W. Kim, E. J. Doyle, T. L. Rhodes, W. A. Peebles, C. L. Rettig and, N. C. Luhmann, Rev. Sci. Instrum. **68**, 466 (1997).

[4] A. G. Stove, IEE proceedings F (Radar and Signal Processing), **139**, 343 (1992).

[5] I. H. Hutchinson, Principles of Plasma dignostics, Cambridge University Press, Cambridge (1987), p125.

[6]Seong-Heon Seo, Jinhyung Park, H. M. Wi, W. R. Lee, H. S. Kim, T. G. Lee, Y. S. Kim, Jin-Seob Kang, M. G. Bog, Y. Yokoda and A. Mase, Rev. Sci. Instrum. **84**, 084702 (2013)

[7]F. Clairet, S. Heuraux, C. Bottereau, D. Molina, L. Ducobu, F. Leroux, and A. Barbuti, Rev. Sci. Instrum. **81**, 10D903 (2010)

[8]Seong-Heon Seo and Dong Ju Lee, Rev. Sci. Instrum. **77**, 045103 (2006).

[9]Seong-Heon Seo, Trans. Fusion Sci. Technol. **55**, 176 (2009).

[10]Seong-Heon Seo and K. D. Lee, Rev. Sci. Instrum. **83**, 10E342 (2012).

[11]Seong-Heon Seo and Dong Keun Oh, Rev. Sci. Instrum. **87**, 11E710 (2016).

[12]See https://siversima.com/wp-content/uploads/VO3262P00-Data-Sheet.pdf for information.