Modelling of the power response in Doppler reflectometry

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1. Introduction

Doppler reflectometry is an experimental technique widely used for the characterization of density fluctuations and flows in fusion plasmas. It is based on the backscattering of a probing microwave beam off the plasma density fluctuations δn. The beam is sent obliquely into the plasma defining an angle θ, with respect to normal incidence. As the beam propagates into the plasma, the refraction index takes on its minimum value at the cutoff layer. These make the diagnostic sensitive to the scattering satisfying the Bragg's condition at a specific perpendicular wavenumber $k_\perp$ and in a well localized radial region.

The frecuency of the measured backscattered signal is Doppler shifted. From the frequency shift f_D the plasma velocity is obtained $v_\perp = 2\pi f_D/k_\perp$ (phase velocity is neglected). The backscattered power P gives a $k_\perp$-resolved turbulence level measurement,

$$P \propto S(k_\perp) = (\delta n_{\mathrm{rms}})^2 \hat{S}(k_\perp), \tag{1}$$

where δn_{rms} is the $k_\perp$-integrated density turbulence level and $\hat{S}(k_\perp)$ is the normalized wavenumber spectrum. By scanning properly the probing beam parameters, DR provides $v_\perp$ profiles and $k_\perp$-spectra [1,2]. Furthermore a second DR channel can be used in order to study the radial correlation length of the turbulence [3].

Nevertheless for some experimental conditions, deviations of the measured $k_\perp$-spectra have been observed in experiments and simulations [2]. A strong flattening of the measured $k_\perp$-spectra is observed when the diagnostic response is non-linear [4]. Recent results from two-dimensional full-wave (2DFW) simulations and physical optics (PO) modelling have characterized in detail the linearity of the response in Doppler reflectometry and the effect of non-linearities in a simple configuration [5]. However, the modelling still has to be applied to more realistic cases in order to give a better interpretation of the experimental data. Small deviations of the measured $k_\perp$-spectrum respect to the input one have been also observed in simulations where non-linear effects are negligible [2,6]. This effect is known as *scattering efficiency*. The effect was studied in Ref. 5 for simple geometries and a correction formula was proposed. However it still has to be applied and tested in more realistic conditions.

The power response in Doppler reflectometry is studied in detail here. First, 2DFW simulation results are presented and a more complete characterization of the power response is given. Later the results of the PO model from Ref. 5 are discussed and extended to more general $k_\perp$-spectra. At the end the scattering efficiency effect is discussed and a correction formula is applied.

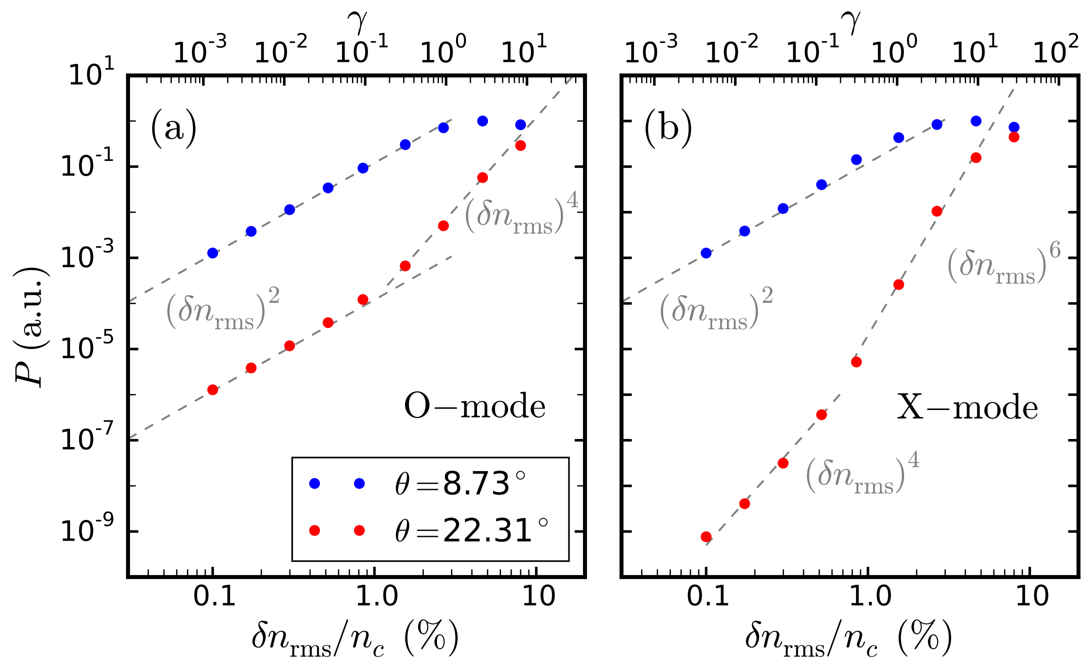

Fig. 1: Backscattered power P as a function of the turbulence level $\delta n_{\mathrm{rms}}/n_c$ for (a) O- and (b) X-mode. 2DFW data are shown for two θ values. Different scalings with δn_{rms} are indicated with dashed lines. The upper axis shows the nonlinearity factor γ.

2. Two-dimensional full-wave modelling

The power response in Doppler reflectometry is investigated using two-dimensional full-wave simulations (2DFW) in a slab geometry using the code described in Ref. 6. Ordinary(O) and extraordinary(X) mode simulations with probing frequencies $f_0 = 60.2$ and 94.4 GHz are performed, respectively. The background density profile is linear and the cut-off is located at 5 cm from the plasma boundary with a density $n_c = 4.5 \cdot 10^{19}$ m^{-3}. A uniform magnetic field with a strength of 2 T is applied. The beam waist is $w = 2.35$ and 1.66 cm for O- and X-mode, respectively. These parameters correspond to typical ASDEX Upgrade conditions. Synthetic Gaussian density turbulence is added to the background density. The normalization of the turbulence field is chosen to set the required turbulence level given by $\delta n_{\mathrm{rms}}/n_c$. The input $k_\perp$-spectrum has a perpendicular correlation length of 0.54 cm.

A scan of the turbulence level is performed for different probing beam angles θ, which set the probed $k_\perp = 2k_0 \sin\theta$. Fig. 1 shows the backscattered power P as a function of the turbulence level for (a) O- and (b) X-mode polarizations. The colours indicate different θ values. The dashed lines indicate different scalings with the turbulence level; $(\delta n_{\mathrm{rms}})^2$, $(\delta n_{\mathrm{rms}})^4$ and $(\delta n_{\mathrm{rms}})^6$. The upper axis shows the nonlinearity parameter γ, which has been introduced in the framework of conventional reflectometry theory [4]. It is proportional to $(\delta n_{\mathrm{rms}}/n_c)^2$ and depends on other experimental parameters, i.a. density profile, probing frequency and polarization.

For small $\delta n_{\rm rms}/n_c$ and small θ (blue points), P increases proportionally to $(\delta n_{\rm rms})^2$. This corresponds to the *linear regime* following the linear response given by Eq. 1. For large $\delta n_{\rm rms}$, P reaches a maximum and then decreases slightly. In this *saturation regime*, non-linear phenomena such as multiple and non-coherent scattering take place. The saturation regime leads to an underestimation of the turbulence level. These two regimes have been already observed and reported in literature [6]. The transition between linear and saturation regime at $\theta = 8.7°$ appears at $\gamma \approx 1$ as predicted by conventional reflectometry theory [4].

Nevertheless for large θ (red points) and intermediate $\delta n_{\rm rms}/n_c$, there is a regime in which P increases faster with $\delta n_{\rm rms}$ than expected in the linear regime. The deviations from Eq. 1 are a clear evidence of the non-linearity. Indeed the $(\delta n_{\rm rms})^4$ and $(\delta n_{\rm rms})^6$ scalings indicated in Fig. 1 suggest double and triple scattering events, respectively. This regime has been called *enhanced power response regime* [5] given that it leads to an overestimation of the turbulence level. The enhanced power response has been also observed in other 2DFW simulations using a realistic plasma geometry and a Kolmogorov-like $k_\perp$-spectrum [7].

The $k_\perp$-spectrum is computed in Fig. 2 by plotting the backscattered power P as a function of $k_\perp$ for X-mode data. The different colours represent various $\delta n_{\rm rms}/n_c$ values. The black line depicts the input $k_\perp$-spectrum computed from the turbulence field δn. The spectra are scaled with $(\delta n_{\rm rms})^{-2}$ in order to compare the different cases. For a small turbulence level (blue line) in the linear regime, P gives a good measurement of the $k_\perp$-spectrum. A slight deviation from the input spectrum is observed, this is due to the scattering efficiency and will be discussed later.

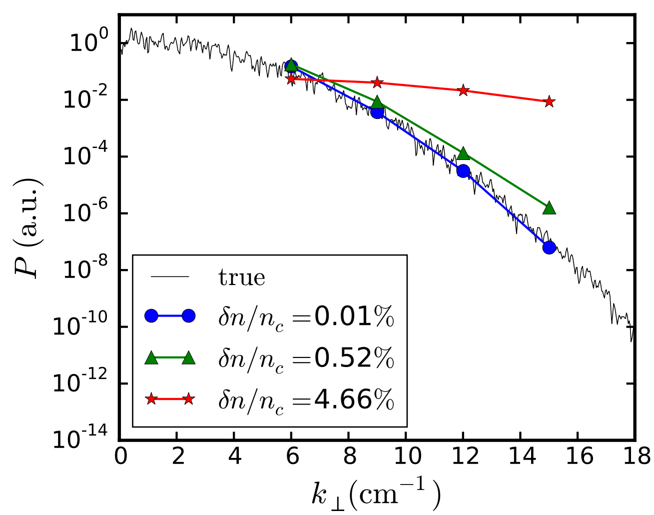

Fig. 2: Scaled $k_\perp$-spectra from 2DFW simulations in X-mode polarization. The different colours represent different turbulence levels. The black line depicts the input spectrum.

For an intermediate turbulence level (green line), the signal at large $k_\perp$ is enhanced and overestimates the spectral density while the signal at small $k_\perp$ is still linear. In this case the enhanced power response is responsible of the flattening of the spectrum at large $k_\perp$. For a large turbulence level (red line), saturation sets in and causes an underestimation of the power for small $k_\perp$, contributing further to the flattening of the spectrum.

3. Physical optics analysis

The physical optics model (PO) is a simple tool originally developed for the study of the scattering of waves by rough surfaces. It has been applied for simulating the scattering process relevant for Doppler reflectometry [5,8]. Although the application of PO modelling to Doppler reflectometry requires strong assumptions, i.a. the good localization of the scattering at the cutoff, it has been able to reproduce several effects observed in 2DFW simulations and experiments. Indeed the physical optics model is a useful tool for studies of the power response: The mathematical expression provided by PO allows analytical estimations leading to a better understanding of the scattering process behind the Doppler reflectometry [5]. The simplicity of the model allows a fast numerical integration, making possible broad and fine scans of the relevant parameters. This is used in this paper in order to obtain a useful representation of the different regimes and its dependence on $k_\perp$ and γ.

In Ref. 5 the PO model reproduces the power response of the already presented 2DFW simulations. Here the PO results for the X-mode case are depicted with solid lines in Fig. 3, 2DFW data are plotted as points. Four θ values are shown. The PO model is able to reproduce linear, enhanced and saturation regimes in good agreement with 2DFW simulations. The vertical displacement of the PO data with respect to the 2DFW data is related to the scattering efficiency and will be discussed in the next section.

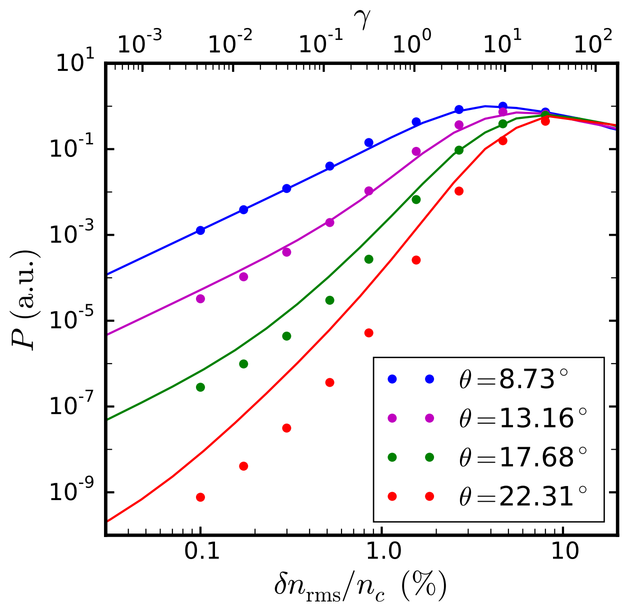

Fig. 3: *Backscattered power P as a function of the turbulence level $\delta n_{\rm rms}/n_c$ for different θ values. X-mode 2DFW data are depicted as points and PO result as solid lines.*

The transition between the different regimes is smooth. Here the saturation is taken at the turbulence level where P has a maximum ($\mathrm{d}P/\mathrm{d}(\delta n_{\rm rms}) = 0$), because from this point on the measured power does not increase with the turbulence level. Taking into account that the enhanced power response yields an overestimation of the spectral intensity, a transition to a *times two* $\times 2$ enhanced response is regard. It occurs at the turbulence level where P is overestimated by a factor of two with respect to the linear response expectation. A $\times 8$ enhanced response with P eight times overestimated is also considered [9].

These definitions allow to illustrate the power response dependence on the turbulence level, or equivalently γ, and on the probed $k_\perp$. In Fig. 4 the linear (green), $\times 2$ (yellow), $\times 8$ (orange) enhanced, and saturation (red) regimes are depicted on the $(k_\perp, \gamma)$-plane for the Gaussian (a) and a flat k-spectrum (b).

For the Gaussian case, a direct transition from linear to saturation regime is observed at small $k_\perp$. In contrast, a strong enhanced power response appears for intermediate turbulence levels and large $k_\perp$. The transition to enhanced power response shifts to smaller γ when $k_\perp$ increases. On the other hand, the flat $k_\perp$-spectrum case depicted in Fig. 4b is radically different from the Gaussian one. A direct transition from linear

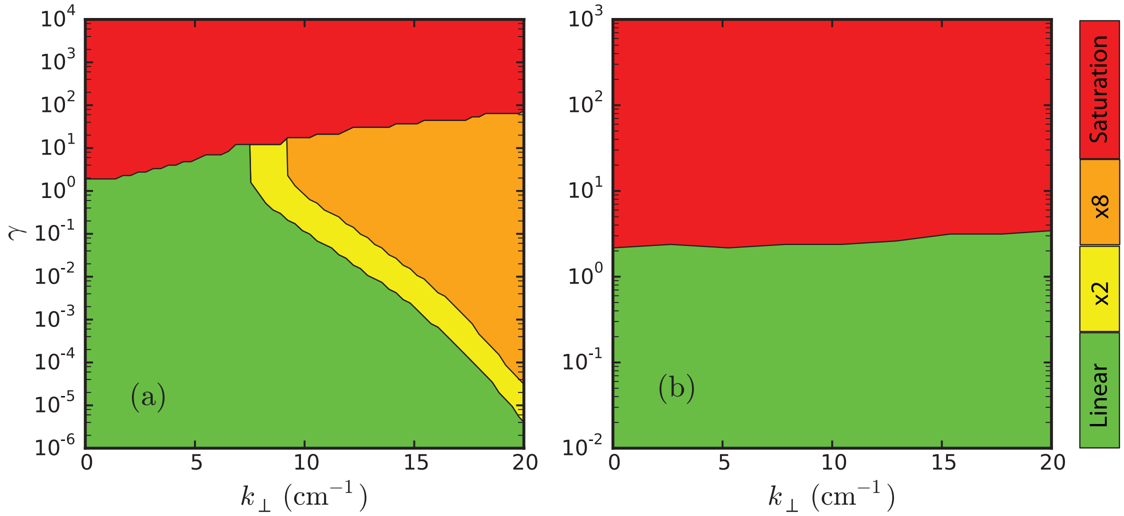

Fig. 4: *Power response map for (a) a Gaussian and (b) a flat k-spectrum. The red colour indicates saturation, yellow* $\times 2$ *and orange* $\times 8$ *enhanced, and green linear regimes.*

to saturation is observed for all $k_\perp$ at approximately the same γ. The absence of the enhanced power response regime in the flat spectrum case explains why it has not been observed in previous works [6,10], where either a completely or locally (around the probed $k_\perp$) flat spectrum was used.

A more realistic Kolmogorov-like $k_\perp$-spectrum is also studied. It has a constant spectral intensity up to 3 cm^{-1} followed by a spectral drop with $k_\perp^{-4}$ (see next section). The power response is represented in Fig. 5. In this case the linear, enhanced and saturation regimes appear. However, the exact dependence is different from the Gaussian case. For $\gamma \ll 1$ the response is linear for every $k_\perp$, while for $\gamma \approx 10$ saturation is reached for small $k_\perp$ while large $k_\perp$ are enhanced. This is in agreement with X-mode experimental spectra, which are flat at low $k_\perp$ and exhibits an underestimated spectral drop at large $k_\perp$ [2,9].

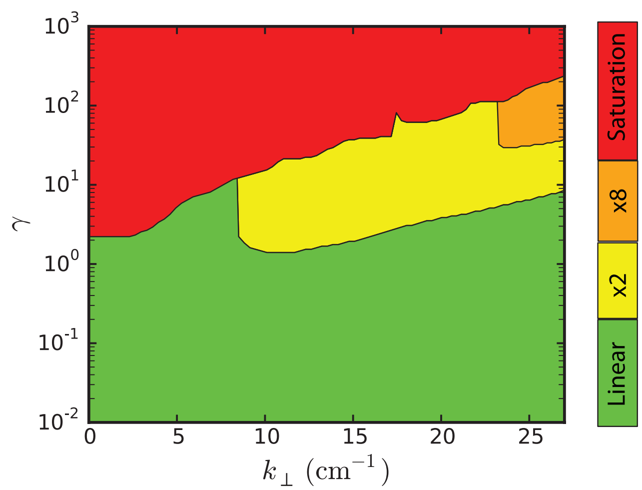

Fig. 5: *Power response map for a Kolmogorov-like* $k_\perp$*-spectrum. The red colour indicates saturation, yellow* $\times 2$ *and orange* $\times 8$ *enhanced, and green linear regimes.*

The three cases show that the spectral shape plays an important role in determining the linearity of the response. Furthermore, the proposed representation of the power response is useful for experimental data analysis. If the turbulence is estimated from other diagnostics, simulations or by reasonable assumptions, it is possible to restrict the γ value and localize the measurements in the $(k_\perp, \gamma)$-plane. This gives information on the $k_\perp$ range where the response is expected to be linear, enhanced or saturated [9].

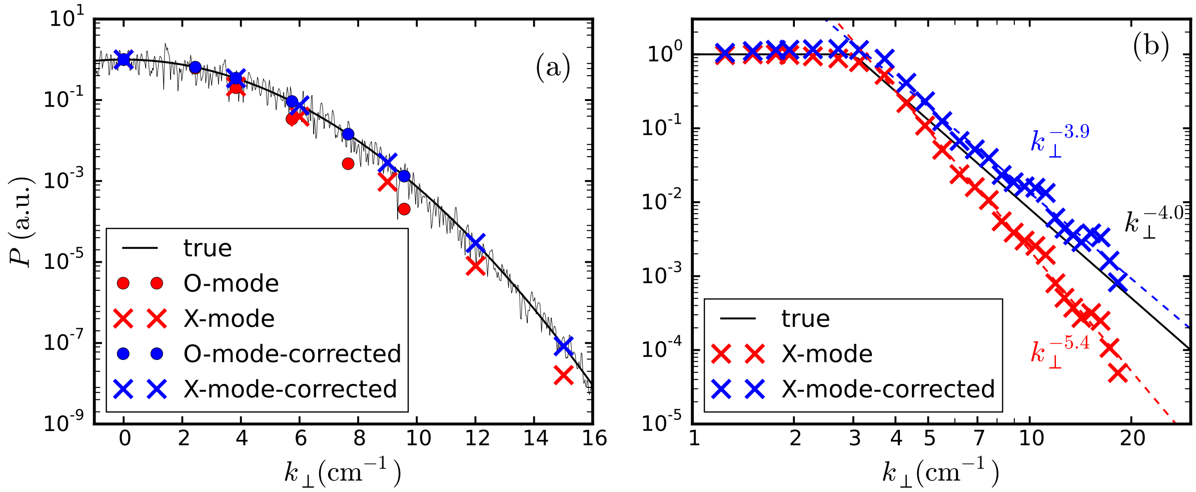

Fig. 6: $k_\perp$-spectra for the (a) Gaussian and (b) Kolomogorov-like case. The input spectra are plotted with black lines and 2DFW data in the linear regime with red symbols. The blue symbols show 2DFW data after applying the scatteing efficiency correction from Eq. 2.

4. Scattering efficiency

Although non-linear effects can distort strongly the measured $k_\perp$-spectra, there are also small deviations from the input spectrum in the linear regime. The detailed propagation of the probing beam for different angles of incidence, produces a slight sensitivity loss at large $k_\perp$ [5].

The scattering efficiency effect is observed by comparing the input $k_\perp$-spectra with 2DFW simulations in the linear regime. Fig. 6 shows the input spectra with black lines and the 2DFW data as red symbols for the (a) Gaussian and (b) Kolmogorov-like cases. The simulations of the Kolmogorovo-like case were performed using the realistic ASDEX Upgrade geometry and are described in Ref. 7. The blue symbols show data corrected as explained below. For both cases the 2DFW $k_\perp$-spectrum exhibits a faster spectral drop as the input spectrum. For the Kolmogorov case the spectrum is flat for small $k_\perp$. However, the spectral index for large $k_\perp$ (-5.4) overestimates the true one (-4.0).

A correction formula accounting for the scattering efficiency has been proposed in Ref. 5. Spectra in the liner regime can be corrected using

$$P_{\text{corrected}} = P\left[1 + \left(\frac{L^{\text{ref}}}{8k_0^2}\right)^{2/3} k_\perp^2\right], \tag{2}$$

where L^{ref} is the gradient scale length of the refractive index and k_0 is the probing beam vacuum wavenumber. The correction factor is applied to the 2DFW data. The results are shown in Fig. 6 by blue symbols. A better agreement with the input $k_\perp$-spectrum is observed in both cases. In particular the corrected spectral index for the Kolmogorov case (-3.9) is in good agreement with the true one.

5. Conclusions

The power response in Doppler reflectometry has been studied in detail by means of two-dimensional full-wave simulations and the physical optics model. The appearance of linear, enhanced power response and saturation regimes has been been studied for different input $k_\perp$-spectra. It has been found that the spectral shape plays a key role for determining the linearity of the power response in Doppler reflectometry measurements. The physical optics analysis can be used to assess the linearity of experimental measurements provided the non-linearity parameter γ can be estimated.

The scattering efficiency has been investigated in slab and realistic plasma geometries with a Gaussian and a Kolmogorov like $k_\perp$-spectrum, respectively. After applying the correction factor from Eq. 6 to two-dimensional full-wave $k_\perp$-spectra, a better agreement with the input spectra is obtained. The correction factor can be applied to experimental data if the power response is linear.

5 6. References

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